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Eigenvalue Analysis of Double-span Timoshenko Beams by Pseudospectral Method

Jinhee Lee*

Department of Mechano-Informatics, Hongik University, Chochiwon, Yeonki-kun, Choongnam 339-701, Korea

The pseudospectral method is applied to the free vibration analysis of double-span Timoshenko beams. The analysis is based on the Chebyshev polynomials. Each section of the double-span beam has its own basis functions, and the continuity conditions at the intermediate support as well as the boundary conditions are treated separately as the constraints of the basis functions. Natural frequencies are provided for different thickness-to-length ratios and for different span ratios, which agree with those of Euler-Bernoulli beams when the thickness-to-length ratio is small but deviate considerably as the thickness-to-length ratio grows larger.

Key Words : Eigenvalue Analysis, Double-span Timoshenko Beam, Pseudospectral Method, Chebyshev Polynomials

1. Introduction

The Euler-Bernoulli beam theory neglects the effect of the transverse shear strain of beam bending because of the assumption that the plane cross-sections perpendicular to the axis of the beam remain plane and perpendicular after deformation. The Euler-Bernoulli beam theory can give excellent solutions to the vibration analysis of slender beams. Beams in real practice, however, may have appreciable thickness where the transverse shear and rotary inertia are not negligible as assumed in the Euler-Bernoulli beam theory. As the result the Timoshenko beam theory that takes the transverse shear and the rotary inertia into consideration has gained more popularity.

Research on beam vibration can be divided into three categories. Firstly, there exist exact solutions only for a restricted number of simple cases. Secondly, studies of semi-analytic solutions are available. Finally, there are the most widely used discretization methods such as the finite element method and the finite difference method. As it is more useful to have analytical results than to resort to numerical methods, most efforts focus on developing efficient semi-analytic solutions.

Multi-span beams are frequently used in many mechanical and civil engineering applications such as the rail systems and the bridges. Gorman computed the natural frequencies of double-span Euler-Bernoulli beams by proposing local solutions for each span and by matching the continuity conditions at the intermediate support (Gorman, 1974). The study on the free vibration of multi-span Euler-Bernoulli beams also has been carried out by various methods such as the finite element method (Hayashikawa and Watanabe, 1985), the Green function method (Kukla, 1991), and the transfer matrix method (Hosking et al., 2004). The free vibration analysis of multi-span beams based on the Timoshenko theory has been investigated using various methods such as Rayleigh-Ritz method (Zhou, 2001)

* Corresponding Author,
E-mail : eng213@naver.com
TEL : +82-42-860-2589; **FAX** : +82-41-862-2664
Department of Mechano-Informatics, Hongik University, Chochiwon, Yeonki-kun, Choongnam 339-701, Korea. (Manuscript **Received** March 22, 2005; **Revised** July 13, 2005)

and the transfer matrix method (Lin and Chang, 2005). Also the response of multi-span beams subjected to moving loads or masses is studied extensively (Cai et al., 1988; Chatterjee et al., 1994; Wang, 1997).

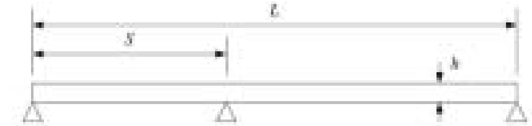
The pseudospectral method can be considered to be a spectral method that performs a collocation process. As the formulation is straightforward and powerful enough to produce approximate solutions close to exact solutions, this method has been highly successful in many areas such as turbulence modeling, weather prediction and non-linear waves (Boyd, 1989). Even though this method can be used for the solution of structural mechanics problems, it has been largely unnoticed by the structural mechanics community, and few articles are available where the pseudospectral method has been applied. Recently it has been successfully applied to the eigenvalue problems of Timoshenko beams and Mindlin plates (Lee, 1998; 2002; 2003a; 2003b; 2003c; 2004; Lee and Schultz, 2004). In the present work, the pseudospectral method is applied to the free vibration analysis of double-span Timoshenko beams.

2. Formulations of Double-span Timoshenko Beams

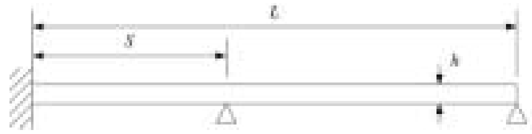
Consider a uniform beam of length L, which is either pinned or clamped at the ends and has a roller support at an intermediate location $x=S$ as depicted in Fig. 1. The equations of motion of the beam in the intervals of $0<x<S$ and $S<x<L$ are given by

$$EI\frac{d^2\theta}{dx^2}+\alpha hG\left(\frac{dw}{dx}-\theta\right)=-\omega^2\rho I\theta$$
$$\alpha hG\frac{d}{dx}\left(\frac{dw}{dx}-\theta\right)=-\omega^2\rho hw \qquad (1)$$

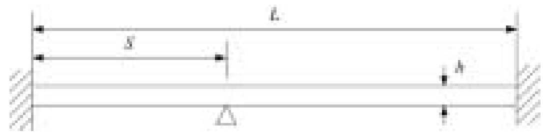
where θ, w and ω are the lateral deflection, the rotation of the normal line and the natural frequency, respectively. E and G are Young's modulus and the shear modulus, α is the shear correction factor, h is the thickness of the beam, I is the second moment of area, and ρ is the density.



(a) Pinned-pinned-pinned support



(b) Clamped-pinned-pinned support



(c) Clamped-pinned-clamped support

Fig. 1 Beam geometry and support conditions

The boundary conditions are either

$$\textit{clamped}:\ \theta=0,\ w=0$$
$$\text{or} \qquad (2)$$
$$\textit{pinned}:\ \frac{d\theta}{dx}=0,\ w=0$$

at $x=0$ and at $x=L$. The continuity conditions at $x=S$ are represented as follows:

$$w(x=S^-)=0$$
$$w(x=S^+)=0$$
$$\theta(x=S^-)=\theta(x=S^+) \qquad (3)$$
$$\frac{d\theta}{dx}(x=S^-)=\frac{d\theta}{dx}(x=S^+)$$

It is convenient to introduce normalized variables z_1 and z_2 such that each of the section between the supports is represented by

$$z_1=\frac{2x-S}{S}\in[-1,\ 1]\ \text{for}\ (0\leq x\leq S)$$
$$z_2=\frac{2x-S-L}{L-S}\in[-1,\ 1]\ \text{for}\ (S\leq x\leq L) \qquad (4)$$

The governing equations (1) can be rewritten as

$$EI\left(\frac{2}{S}\right)^2\theta''-\alpha hG\theta+\alpha hG\frac{2}{S}\theta'=-\omega^2\rho I\theta$$
$$-\alpha hG\frac{2}{S}\theta'+\alpha hG\left(\frac{2}{S}\right)^2 w''=-\omega^2\rho hw \quad (5)$$
$$(-1<z_1<1)$$

and

$$EI\left(\frac{2}{L-S}\right)^2\theta^{\dagger\dagger}-\alpha hG\theta+\alpha hG\frac{2}{L-S}\theta^{\dagger}=-\omega^2\rho I\theta$$
$$-\alpha hG\frac{2}{L-S}\theta^{\dagger}+\alpha hG\left(\frac{2}{L-S}\right)^2 w^{\dagger\dagger}=-\omega^2\rho hw \quad (6)$$
$$(-1<z_2<1)$$

where $'$ and $\dagger$ represent the differentiations with respect to z_1 and z_2, respectively. The series expansions of the exact solutions $\theta(x)$ and $w(x)$ have infinite numbers of terms. In this study, however, the dependent variables are approximated by the partial sums as follows:

$$\theta(x)\cong\bar{\theta}(z_1)=\sum_{k=1}^{K+2}a_kT_{k-1}(z_1)$$
$$w(x)\cong\bar{w}(z_1)=\sum_{k=1}^{K+2}b_kT_{k-1}(z_1) \quad (-1<z_1<1) \quad (7)$$

and

$$\theta(x)\cong\bar{\theta}(z_2)=\sum_{n=1}^{N+2}c_nT_{n-1}(z_2)$$
$$w(x)\cong\bar{w}(z_2)=\sum_{n=1}^{N+2}d_nT_{n-1}(z_2) \quad (-1<z_2<1) \quad (8)$$

where a_k, b_k, c_n and d_n are the expansion coefficients and T_{n-1} is the Chebyshev polynomial of the first kind of degree of $n-1$. Mikami and Yoshimura suggested an efficient way to handle the boundary conditions by adopting less collocation points than the number of expansion terms (Mikami and Yoshimura, 1984).

Expansions (7) and (8) are substituted into Eqs. (5) and (6) and are collocated at the Gauss-Lobatto collocation points

$$\xi_i=-\cos\frac{\pi(2i-1)}{2K}\ (i=1,2,\cdots,K)\ \text{for}\ (-1<z_1<1)$$
$$\eta_j=-\cos\frac{\pi(2j-1)}{2N}\ (j=1,2,\cdots,N)\ \text{for}\ (-1<z_2<1) \quad (9)$$

to yield

$$\sum_{k=1}^{K+2}\left[a_k\left\{\frac{4EI}{S^2}T''_{k-1}(\xi_i)-\alpha hGT_{k-1}(\xi_i)\right\}+b_k\frac{2\alpha hG}{S}T'_{k-1}(\xi_i)\right]$$
$$=-\omega^2\rho I\sum_{k=1}^{K+2}a_kT_{k-1}(\xi_i)$$
$$\sum_{k=1}^{K+2}\left\{-a_k\frac{2\alpha hG}{S}T'_{k-1}(\xi_i)+b_k\frac{4\alpha hG}{S^2}T''_{k-1}(\xi_i)\right\} \quad (10)$$
$$=-\omega^2\rho h\sum_{k=1}^{K+2}b_kT_{k-1}(\xi_i)$$
$$(i=1,2,\cdots,K)$$

and

$$\sum_{n=1}^{N+2}\left[c_n\left\{\frac{4EI}{(L-S)^2}T^{\dagger\dagger}_{n-1}(\eta_j)-\alpha hGT_{n-1}(\eta_j)\right\}+d_n\frac{2\alpha hG}{L-S}T^{\dagger}_{k-1}(\eta_j)\right]$$
$$=-\omega^2\rho I\sum_{n=1}^{N+2}c_nT_{n-1}(\eta_j)$$
$$\sum_{n=1}^{N+2}\left\{-c_n\frac{2\alpha hG}{L-S}T^{\dagger}_{n-1}(\eta_j)+d_n\frac{4\alpha hG}{(L-S)^2}T^{\dagger\dagger}_{n-1}(\eta_j)\right\} \quad (11)$$
$$=-\omega^2\rho h\sum_{n=1}^{N+2}d_nT_{n-1}(\eta_j)$$
$$(j=1,2,\cdots,N)$$

Eqs. (10) and (11) can be rearranged in the matrix form

$$[H]\{\delta\}+[H^*]\{\delta^*\}$$
$$=\omega^2([F]\{\delta\}+[F^*]\{\delta^*\}) \quad (12)$$

where the vectors in Eq. (12) are defined by

$$\{\delta\}=\{a_1\ a_2\cdots a_K\ b_1\ b_2\cdots b_K\ c_1\ c_2\cdots c_N\ d_1\ d_2\cdots d_N\}^T$$
$$\{\delta^*\}=\{a_{K+1}\ a_{K+2}\ b_{K+1}\ b_{K+2}\ c_{N+1}\ c_{N+2}\ d_{N+1}\ d_{N+2}\}^T \quad (13)$$

The total number of equations in Eq. (12) is $2(K+N)$ whereas the total number of unknowns in Eq. (13) is $2(K+N+4)$. The remaining eight equations are obtained from the continuity conditions and the boundary conditions.

Using the expansions (7) and (8), the continuity conditions (3) can be rewritten as

$$\sum_{k=1}^{N+2}b_kT_{k-1}(1)=0$$
$$\sum_{n=1}^{K+2}d_nT_{n-1}(-1)=0$$
$$\sum_{k=1}^{K+2}a_kT_{k-1}(1)=\sum_{n=1}^{N+2}c_nT_{n-1}(-1) \quad (14)$$
$$\frac{1}{S}\sum_{k=1}^{K+2}a_kT'_{k-1}(1)=\frac{1}{L-S}\sum_{n=1}^{N+2}c_nT^{\dagger}_{n-1}(-1)$$

The boundary conditions are either

$$\text{clamped}: \sum_{k=1}^{K+2} a_k T_{k-1}(-1)=0$$
$$\sum_{k=1}^{K+2} b_k T_{k-1}(-1)=0$$

or (15)

$$\text{pinned}: \sum_{k=1}^{K+2} a_k T'_{k-1}(-1)=0$$
$$\sum_{k=1}^{K+2} b_k T_{k-1}(-1)=0$$

at $x=0$, and either

$$\text{clamped}: \sum_{n=1}^{N+2} c_n T_{n-1}(1)=0$$
$$\sum_{n=1}^{N+2} d_n T_{n-1}(1)=0$$

or (16)

$$\text{pinned}: \sum_{n=1}^{N+2} c_n T^{\dagger}_{n-1}(1)=0$$
$$\sum_{k=1}^{N+2} d_k T_{k-1}(1)=0$$

at $x=L$.

The continuity conditions (14) and boundary conditions (15)-(16) can be rearranged in the matrix form

$$[U]\{\delta\}+[V]\{\delta^*\}=\{0\} \tag{17}$$

where $\{0\}$ is a zero vector. Since $\{\delta^*\}$ in Eq. (17) can be expressed as

$$\{\delta^*\}=-[V]^{-1}[U]\{\delta\} \tag{18}$$

the set of equations (12) can be reformulated as

$$\begin{aligned}&([H]-[H^*][V]^{-1}[U])\{\delta\}\\ &=\omega^2([F]-[F^*][V]^{-1}[U])\{\delta\}\end{aligned} \tag{19}$$

The solution of (19) yields the estimate for the natural frequencies and the corresponding mode shapes.

3. Numerical Examples

A preliminary run for the convergence check of the eigenvalues of a double-span Timoshenko beam which has a clamped-pinned-pinned support is carried out for $h/L=0.01$ and $S/L=0.5$, and the results are given in Table 1. The numbers of collocation points which determines the size of the problem change from $K=M=3$ to $K=M=20$. The total number of equations in (10) and (11) is $2(K+N)$, and the size of matrices in equation (19) becomes 80×80 for $K=M=20$. Table 1 clearly shows the rapid convergence nature of the pseudospectral method, where it is readily shown that it requires less than $K=M=10$ for the 4 lowest eigenvalues to converge to 6 significant digits, and less than $K=M=15$ for eigenvalues of the 10 lowest modes to 6 significant digits. The numbers given in Tables 1~4 are the non-dimensionalized frequency parameters β defined as

$$\beta=\sqrt[4]{\rho A\omega^2/EI} \tag{20}$$

where A is the cross sectional area of the beam.

Table 1 Convergence test of the non-dimensionalized frequency parameter β of the double span Timoshenko beam as the number of the collocation points increase (clamped-pinned-pinned support, $\nu=0.3$, $\alpha=5/6$, $h/L=0.01$, $S/L=0.5$)

Mode	$K=N=3$	$K=N=5$	$K=N=10$	$K=N=15$	$K=N=20$
1	6.92346	6.78556	6.78306	6.78306	6.78306
2	9.20854	8.93385	8.91641	8.91641	8.91641
3		13.8677	13.0692	13.0692	13.0692
4		16.6498	15.1408	15.1408	15.1408
5		21.3141	19.3074	19.3059	19.3059
6		24.3702	21.3626	21.3590	21.3590
7			25.5133	25.5046	25.5046
8			27.5497	27.5302	27.5302
9			32.2921	31.6532	31.6532
10			34.9092	33.6454	33.6454

Through out the paper, Poisson's ratio and the shear coefficient of the beam are $\nu=0.3$ and $\alpha=5/6$, respectively.

Computational results for the collocation points $K=M=20$ with pinned-pinned-pinned, clamped-pinned-pinned, and clamped-pinned-clamped supports are given in Tables 2~4, respectively. The natural frequencies are calculated for different thickness-to-length ratios ranging from $h/L=0.005$ to $h/L=0.1$. It is well known that the static and dynamic characteristics of Timoshenko beams approach those of Euler-Bernoulli beams when the thickness of the beams is very small, and the eigenvalues based on the Euler-Bernoulli theory (Gorman, 1974) are given in Tables 2~4 for the purpose of comparison. The results of Tables 2~4 show that the Timoshenko beam results are very close to the Euler-Bernoulli beam results when the thickness-to-length ratio h/L is small, showing that at least three significant digits are identical with the Euler-Bernoulli results in most cases when the thickness-to-length ratio is 0.005. As h/L grows larger, however, the computed eigenvalues show some quantitative differences from those of Euler-Bernoulli beams. The natural frequencies ω in Tables 2~4 increase as h/L increases, even though the frequency parameters β in Tables 2~4 tend to decrease because the second moment of area I grows faster than ω^2 as h/L increases.

It is possible that there might be optimal combinations of K and M, the numbers of the Gauss-Labotto collocation points, when the size of one span is different from the other, however, they are assumed to be the same for the sake of simplicity. It is also shown that the computed

Table 2 Non-dimensionalized frequency parameter β of the double span Timoshenko beam (pinned-pinned-pinned support, $\nu=0.3$, $\alpha=5/6$, $K=N=20$)

S/L	Mode	Classical theory	h/L				
			0.005	0.01	0.02	0.05	0.1
0.1	1	4.22637	4.22591	4.22455	4.21913	4.18246	4.06718
	2	7.63130	7.62983	7.62542	7.60798	7.49282	7.15781
	3	11.0505	11.0469	11.0361	10.9934	10.7217	10.0034
	4	14.4793	14.4718	14.4497	14.3633	13.8364	12.5864
	5	17.9123	17.8990	17.8592	17.7057	16.8169	14.9262
0.2	1	4.61832	4.61794	4.61680	4.61224	4.58106	4.47920
	2	8.39155	8.39000	8.38536	8.36697	8.24478	7.88141
	3	12.1617	12.1576	12.1452	12.0966	11.7863	10.9647
	4	15.7080	15.6997	15.6749	15.5784	14.9926	13.6132
	5	17.8725	17.8574	17.8127	17.6395	16.6350	14.5092
0.3	1	5.13179	5.13136	5.13010	5.12506	5.09060	4.97761
	2	9.27693	9.27513	9.26976	9.24847	9.10743	8.69103
	3	11.7804	11.7760	11.7630	11.7119	11.3851	10.5183
	4	14.2845	14.2769	14.2544	14.1666	13.6316	12.3683
	5	18.4048	18.3907	18.3488	18.1870	17.2526	15.2730
0.4	1	5.78261	5.78210	5.78058	5.77451	5.73309	5.59796
	2	8.76786	8.76607	8.76073	8.73954	8.59896	8.18261
	3	11.3129	11.3091	11.2976	11.2522	10.9627	10.1941
	4	15.7080	15.6997	15.6749	15.5784	14.9926	13.6132
	5	17.3296	17.3158	17.2749	17.1165	16.1984	14.2547
0.5	1	6.28319	6.28265	6.28106	6.27471	6.23136	6.09066
	2	7.85321	7.85163	7.84690	7.82817	7.70352	7.33122
	3	12.5664	12.5621	12.5494	12.4994	12.1813	11.3431
	4	14.1372	14.1294	14.1062	14.0154	13.4611	12.1454
	5	18.8496	18.8352	18.7926	18.6282	17.6810	15.6790

Table 3 Non-dimensionalized frequency parameter β of the double span Timoshenko beam (clamped-pinned-pinned support, $\nu=0.3$, $\alpha=5/6$, $K=N=20$)

S/L	Mode	Classical theory	h/L				
			0.005	0.01	0.02	0.05	0.1
0.1	1	4.25636	4.25557	4.25324	4.24418	4.19078	4.07085
	2	7.67648	7.67446	7.66845	7.64507	7.50420	7.16181
	3	11.1062	11.1019	11.0889	11.0383	10.7343	10.0068
	4	14.5436	14.5352	14.5103	14.4141	13.8498	12.5890
	5	17.9862	17.9716	17.9285	17.7629	16.8312	14.9281
0.2	1	4.67394	4.67338	4.67170	4.66504	4.62108	4.49240
	2	8.46945	8.46759	8.46204	8.44010	8.29731	7.89667
	3	12.2832	12.2785	12.2645	12.2095	11.8646	10.9876
	4	16.0717	16.0620	16.0331	15.9209	15.2512	13.7356
	5	19.6346	19.6168	19.5639	19.3604	18.1949	15.6416
0.3	1	5.21414	5.21357	5.21189	5.20518	5.15993	5.01886
	2	9.47849	9.47628	9.46967	9.44352	9.27248	8.78674
	3	13.3430	13.3370	13.3190	13.2484	12.8034	11.6438
	4	15.0778	15.0667	15.0338	14.9062	14.1554	12.5533
	5	18.5948	18.5792	18.5329	18.3548	17.3471	15.2952
0.4	1	5.92267	5.92200	5.92000	5.91202	5.85800	5.68760
	2	10.1680	10.1651	10.1564	10.1223	9.89863	9.25957
	3	11.7988	11.7936	11.7778	11.7159	11.3304	10.3772
	4	16.1768	16.1668	16.1372	16.0220	15.3361	13.7909
	5	18.7193	18.7003	18.6437	18.4263	17.2019	14.7687
0.5	1	6.78646	6.78561	6.78306	6.77291	6.70440	6.48964
	2	8.92665	8.92408	8.91641	8.88607	8.68790	8.12874
	3	13.0908	13.0854	13.0692	13.0056	12.6079	11.6079
	4	15.1832	15.1726	15.1408	15.0173	14.2814	12.6323
	5	19.3731	19.3562	19.3059	19.1128	18.0245	15.8265
0.6	1	6.92042	6.91939	6.91630	6.90403	6.82124	6.56241
	2	9.05288	9.05058	9.04371	9.01651	8.83870	8.33524
	3	12.4682	12.4624	12.4450	12.3769	11.9505	10.8798
	4	16.2966	16.2863	16.2556	16.1364	15.4279	13.8400
	5	18.1094	18.0924	18.0418	17.8475	16.7477	14.5324
0.7	1	6.20547	6.20461	6.20201	6.19167	6.12179	5.90227
	2	10.1984	10.1957	10.1874	10.1545	9.94041	9.33654
	3	12.1248	12.1194	12.1032	12.0400	11.6436	10.6444
	4	15.2865	15.2764	15.2462	15.1287	14.4279	12.8534
	5	19.4211	19.4028	19.3487	19.1409	17.9699	15.6205
0.8	1	5.57754	5.57682	5.57465	5.56604	5.50770	5.32326
	2	9.33697	9.33447	9.32698	9.29737	9.10373	8.55533
	3	13.0892	13.0832	13.0656	12.9963	12.5637	11.4789
	4	16.4345	16.4239	16.3922	16.2690	15.5378	13.9044
	5	18.3252	18.3075	18.2552	18.0538	16.9146	14.6229
0.9	1	5.09491	5.09416	5.09193	5.08306	5.02342	4.84002
	2	8.48413	8.48191	8.47525	8.44894	8.27728	7.79470
	3	11.9070	11.9020	11.8868	11.8274	11.4550	10.5143
	4	15.3373	15.3275	15.2983	15.1848	14.5071	12.9787
	5	18.7706	18.7537	18.7035	18.5105	17.4200	15.2193

Table 4 Non-dimensionalized frequency parameter β of the double span Timoshenko beam (clamped-pinned-clamped support, $\nu=0.3$, $\alpha=5/6$, $K=N=20$)

S/L	Mode	Classical theory	h/L				
			0.005	0.01	0.02	0.05	0.1
0.1	1	5.12956	5.12842	5.12504	5.11187	5.03279	4.84391
	2	8.53225	8.52942	8.52100	8.48819	8.28899	7.79851
	3	11.9650	11.9591	11.9416	11.8737	11.4677	10.5175
	4	15.4038	15.3930	15.3609	15.2369	14.5204	12.9812
	5	18.8474	18.8292	18.7753	18.5695	17.4344	15.2211
0.2	1	5.63992	5.63898	5.63617	5.62505	5.55167	5.33712
	2	9.42152	9.41866	9.41010	9.37634	9.15918	8.57073
	3	13.2354	13.2288	13.2090	13.1315	12.6552	11.5048
	4	17.0022	16.9894	16.9512	16.8036	15.9445	14.1065
	5	20.3723	20.3497	20.2827	20.0261	18.5905	15.7041
0.3	1	6.30151	6.30046	6.29731	6.28479	6.20116	5.94770
	2	10.5280	10.5245	10.5139	10.4722	10.2044	9.48322
	3	13.9338	13.9255	13.9010	13.8051	13.2161	11.8024
	4	15.7260	15.7134	15.6762	15.5321	14.6974	12.9468
	5	19.6366	19.6165	19.5569	19.3291	18.0715	15.6424
0.4	1	7.14942	7.14808	7.14405	7.12808	7.02152	6.70052
	2	10.6107	10.6067	10.5949	10.5484	10.2503	9.45384
	3	12.7332	12.7264	12.7062	12.6269	12.1398	10.9681
	4	17.2339	17.2204	17.1803	17.0251	16.1249	14.2145
	5	19.0450	19.0238	18.9608	18.7199	17.3909	14.8441
0.5	1	7.85321	7.85163	7.84690	7.82817	7.70352	7.33122
	2	9.46008	9.45680	9.44699	9.40829	9.15909	8.48403
	3	14.1372	14.1294	14.1062	14.0154	13.4611	12.1454
	4	15.7064	15.6938	15.6563	15.5112	14.6624	12.8359
	5	20.4204	20.3985	20.3338	20.0868	18.7318	16.1487

natural frequencies tend to approach those of the single-span beam results as the span ratio S/L approaches either the unity or zero.

4. Conclusions

The pseudospectral method is applied to the free vibration analysis of double-span Timoshenko beams. Although the Rayleigh-Ritz method and the differential quadrature method have been successful in the vibration analysis of Timoshenko beams, there are some drawbacks inherent in these methods. For example, they require a process of constructing either weighting coefficients or characteristic polynomials since there are no readily available formulas. The pseudospectral method, on the other hand, uses simple series expansions such as the Chebyshev polynomials as basis functions. The formulation as well as coding for computation is straightforward because the pseudospectral method undergoes the simple collocation process instead of integration.

Basis functions are assumed for each section of the double-span beam. The continuity conditions at the intermediate support and the boundary conditions are considered as the side constraints, and the set of algebraic equations is condensed so that the number of degrees of freedom of the title problem matches the number of the pseudospectral expansion coefficients.

Numerical examples are provided for various thickness-to-length ratios and span ratios. The results from this method agree with those of Euler-Bernoulli beams when the thickness-to-length ratio is very small, however, deviate considerably as the thickness-to-length ratio grows larger.

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